Teaching Through Problem Solving

Session 1

facilitated by
Kathy Kubota-Zarivnij

GAINS
Growing Accessible Interactive Networked Supports

Overall Learning Goals

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- use curriculum expectations and mathematical processes (i.e., grades 6 to 10) to anticipate a range of solutions to a problem, to understand and describe the mathematics in student solutions, and to judge the appropriateness of problems for teaching/learning
- develop a knowledge package or landscape of clustered curriculum expectations
- use the three-part problem solving lesson design to frame the use of problems for teaching mathematics
- develop strategies and mathematics knowledge for anticipating student responses and understanding students' mathematical thinking
- experience strategies for developing students' mathematical communication through the discourse of a math-talk learning community, teacher recording strategies (blackboard writing – mathematical annotations), and coordination and recording of discussion (bansho)
- vary the structure of the problem for students to practise new learning and to provoke use of some strategies and not others, moving towards strategies that can generalize

KKZ, 2008

Setting the Context
- What Do You Notice?

1. What do you notice about Ma and Pa Kettel's mathematics knowledge, skills, and strategies?
2. How do you think they learned mathematics?

Ma-Pa-Kettel-Math.wmv

Setting the Context
- What Do You Think?

3. What does effective mathematics teaching and learning look, sound, and feel like?
4. What are some challenges in teaching and learning mathematics effectively for adolescent students?

Round Robin

Setting the Context
- Why? Why? Why?

- Why study mathematics from grades 6 to 10?
- Why study number sense and algebra?
- Why study teaching mathematics through problem solving?

Setting The Context

- Suurtamm and Graves, CIIM 2007

Familiarity	Grade 8 Mathematics		Grade 9 Academic Mathematics	
	Grade 7 and 8 teachers	Grade 9 and 10 teachers	Grade 7 and 8 teachers	Grade 9 and 10 teachers
Not at all	3%	20%	24%	2%
A little	5%	36%	33%	5%
Somewhat	18%	36%	35%	18%
Very	75%	8%	8%	76%

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Setting The Context

- Suurtamm and Graves, CIIM 2007

Areas for professional development	Somewhat/A lot		
	Grade 7 and 8 teachers	Grade 9 and 10 teachers	All respon-dents
Teaching through problem solving	84%	75%	78%
Understanding how students learn mathematics	78%	67%	72%
Teaching strategies	76%	66%	71%
Use of manipulatives	79%	57%	67%
Assessment in mathematics	79%	55%	66%
Using group work in mathematics	71%	52%	61%
Facilitating investigations	67%	54%	60%
Use of other computer software (other than GSP)	72%	42%	56%
Use of Geometer's Sketchpad	68%	40%	53%
Use of graphing calculators	52%	42%	46%
Content knowledge for teaching	48%	27%	37%
Other (please specify in comments section on the next page)	30%	24%	27%

Setting The Context
- Ontario Curriculum, 2005

Problem Solving

Problem solving is central to learning mathematics. By learning to solve problems and by learning *through* problem solving, students are given numerous opportunities to connect mathematical ideas and to develop conceptual understanding. Problem solving forms the basis of effective mathematics programs and should be the mainstay of mathematical instruction.

It is considered an essential process through which students are able to achieve the expectations in mathematics, and is an integral part of the mathematics curriculum in Ontario, for the following reasons. Problem solving:

- is the primary focus and goal of mathematics in the real world;
- helps students become more confident in their ability to do mathematics;
- allows students to use the knowledge they bring to school and helps them connect mathematics with situations outside the classroom;
- helps students develop mathematical understanding and gives meaning to skills and concepts in all strands;
- allows students to reason, communicate ideas, make connections, and apply knowledge and skills;

Learning Mathematics *for* Teaching
(Ball, 2005)

- Expert personal knowledge of mathematics is ironically inadequate for teaching elementary students mathematics.
- Effective teachers must be able to work backward from their own mature and compressed mathematics content knowledge and unpack its core elements.
- Effective teachers enable students to access new mathematical ideas accessible to students through their own thinking and construction.
- Effective teachers work with the mathematics that students present, often in an unfinished, yet growing state.

What Does Knowing Mathematics for Teaching … Ball (2005)

THE LITERACY AND NUMERACY SECRETARIAT presents...
in partnership with Curriculum Services Canada

Deborah Ball PartA.wmv

DR. DEBORAH LOEWENBERG BALL

November 2, 2005 webcast

CHAPTERS

1: Intro
2: Purpose and Overview
3: Deborah's Teaching Background and Inquiry
4: What Mathematical Knowledge do Teachers need
5: The Quality of Mathematics Teaching
6: Practice-Based Approach
7: Doing Mathematics as a Teacher
8: Mathematics Knowledge for Teaching
9: Research About Mathematics Knowledge for Teaching
10: Learning Mathematics for Teaching
11: Outro

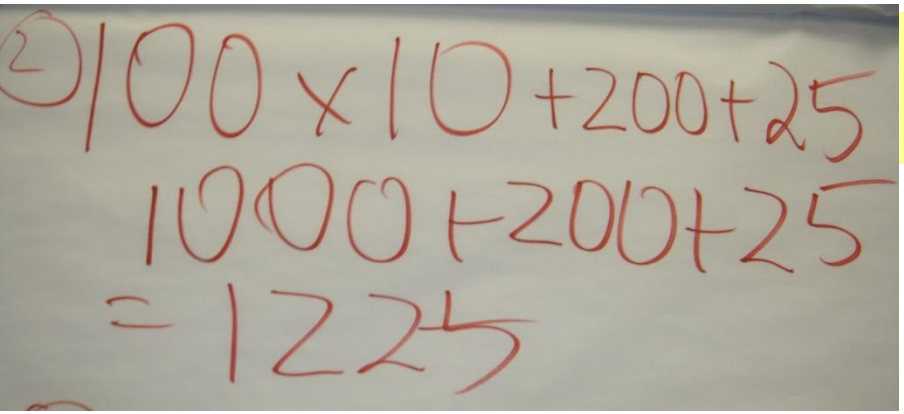
PLAY

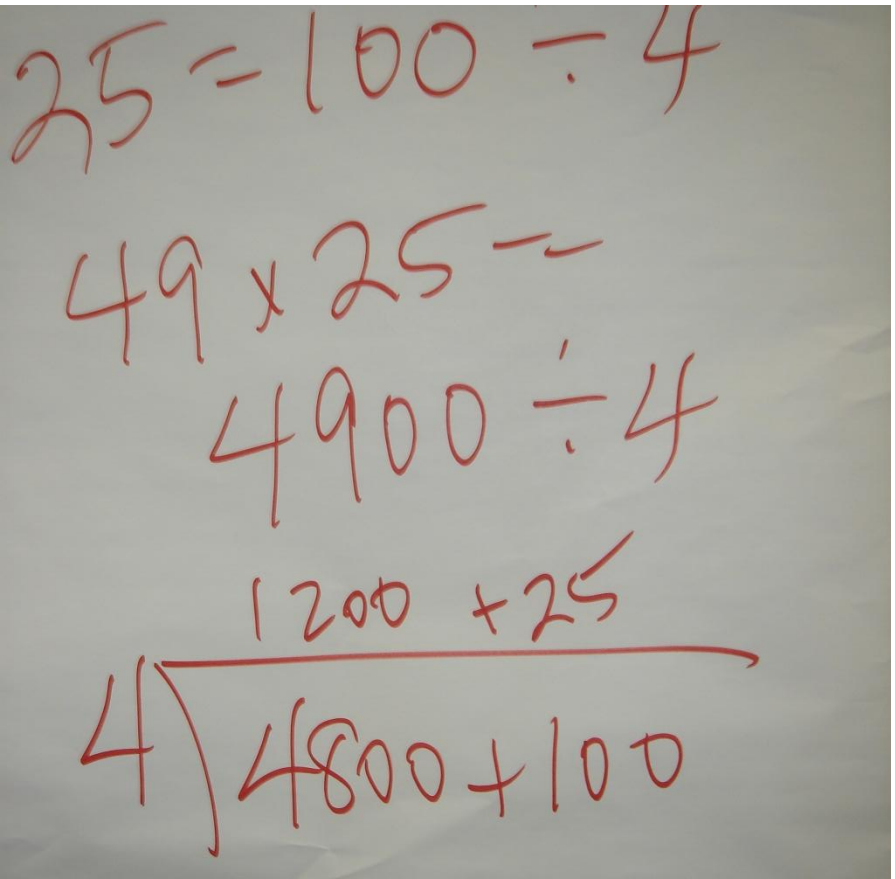
What Different Solutions are Possible for the Product 1225

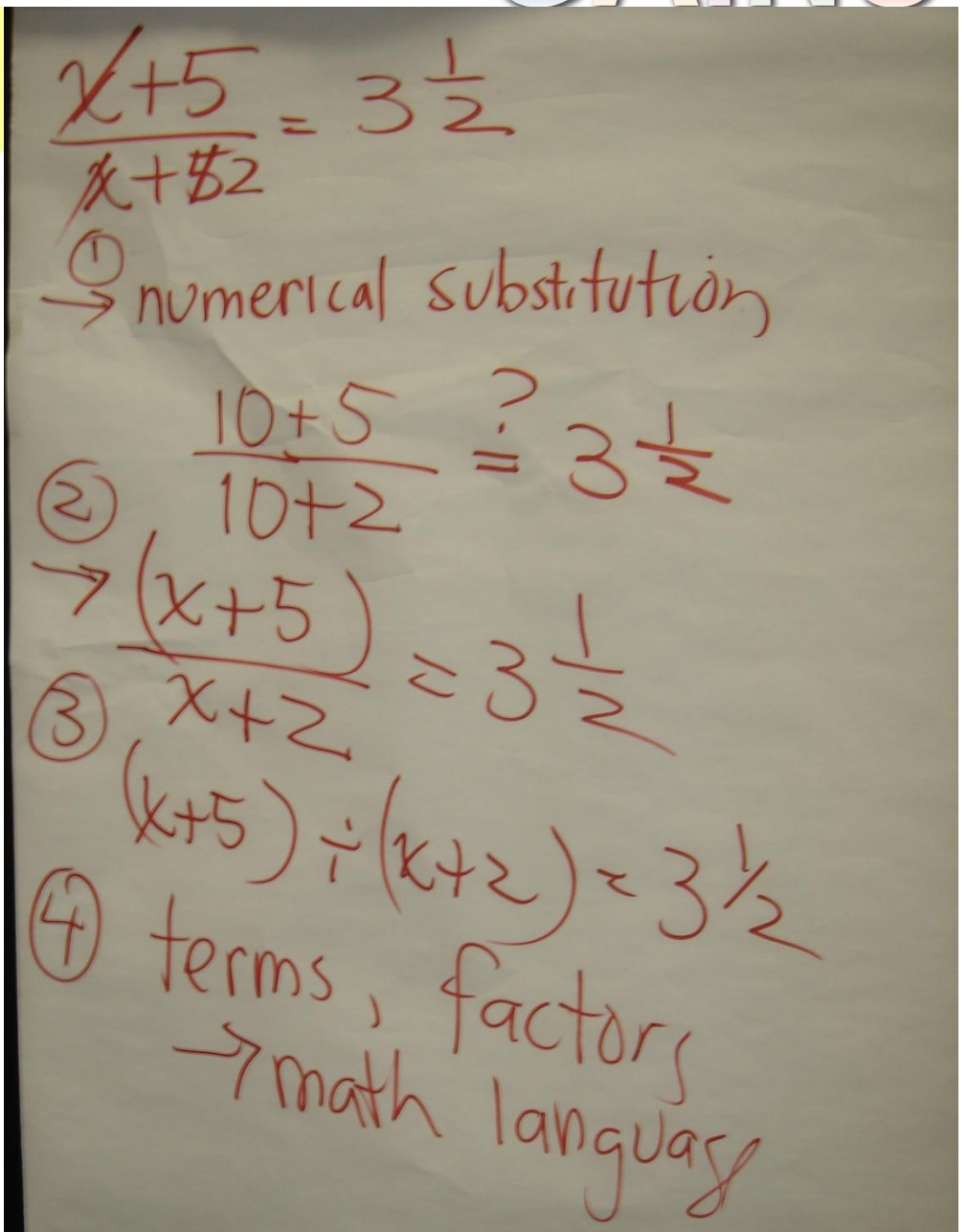
Multiply:

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline \end{array}$$

GAINS


② 100 x 10 + 200 + 25
1000 + 200 + 25
= 1225


25 = 100 ÷ 4
49 x 25 =
4900 ÷ 4
1200 + 25
4 | 4800 + 100


x+5 / x+2 = 3 1/2
① → numerical substitution
10+5 / 10+2 ≟ 3 1/2
② → (x+5) / x+2 = 3 1/2
③ (x+5) ÷ (x+2) = 3 1/2
④ terms, factors
→ math language

Knowledge of Mathematics *for* Teaching

Anticipating a Range of Solutions

Knowing Multiplication

Multiply:

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline \end{array}$$

Decompose - Partial Products

49 x 25 => 40 x 20 = 800

=> 9 x 20 = 180

=> 40 x 5 = 200

=> 9 x 5 = 45

1225

Friendly Numbers

25 x 4 = 100

100 x 49 = 4900

4900 ÷ 4 =>4000÷4 =**1000**

800 ÷ 4 = **200**; 100 ÷4= **25**

1000 + **200** + **25** = 1225

Using Coins - Quarters

4 x 25¢ = 100¢

10 x 100¢ = **1000¢**

So, 40 x 25¢= 1000¢

9 x 25¢ = 8 x 25¢ + 1 x 25¢

= ((4 x 25¢) x 2)+25¢ = **225¢**

1000¢ + **225¢** = 1225¢

What Does Knowing Mathematics for Teaching … Ball (2005)

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PLAY

Knowledge of Mathematics *for* Teaching

Analyzing Student Errors to Understand Math Thinking

How was each answer produced?

What might lead a student to make these errors?

Group 1

(a)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 405 \\ 108 \\ \hline 1485 \end{array}$$

Group 2

(b)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 225 \\ 100 \\ \hline 325 \end{array}$$

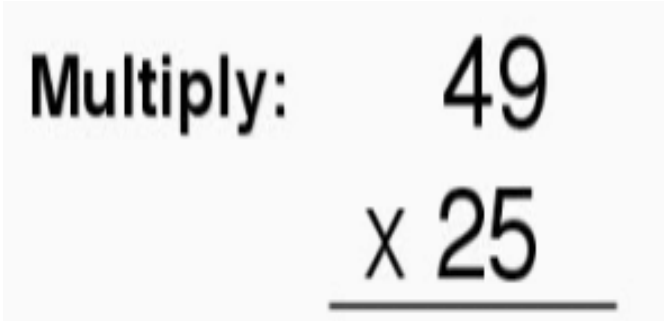
Group 3

(c)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 1250 \\ 25 \\ \hline 1275 \end{array}$$

Turn and Talk

Three-Part Problem Solving Lesson
Before ... During ... After

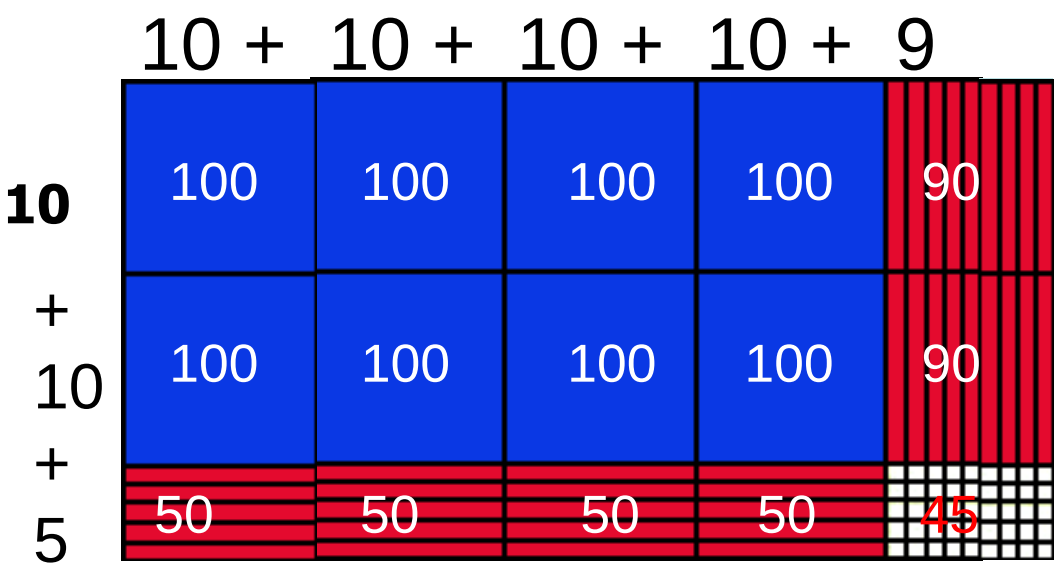


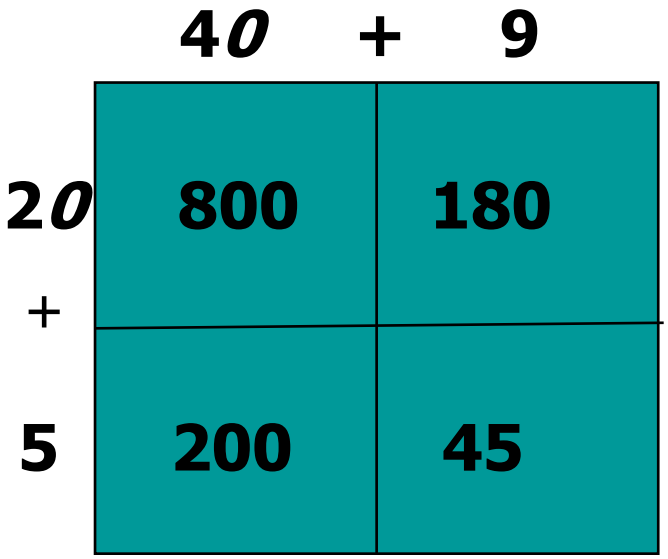
- Before (Activation)
- During
- After

Learning Goal:
Multiply 2 digit by
2 digit numbers using
different strategies

What Does It Look Like?

Whole Numbers … 49 x 25 = ?



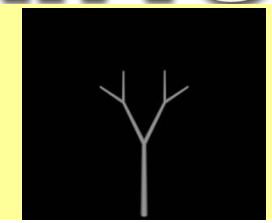


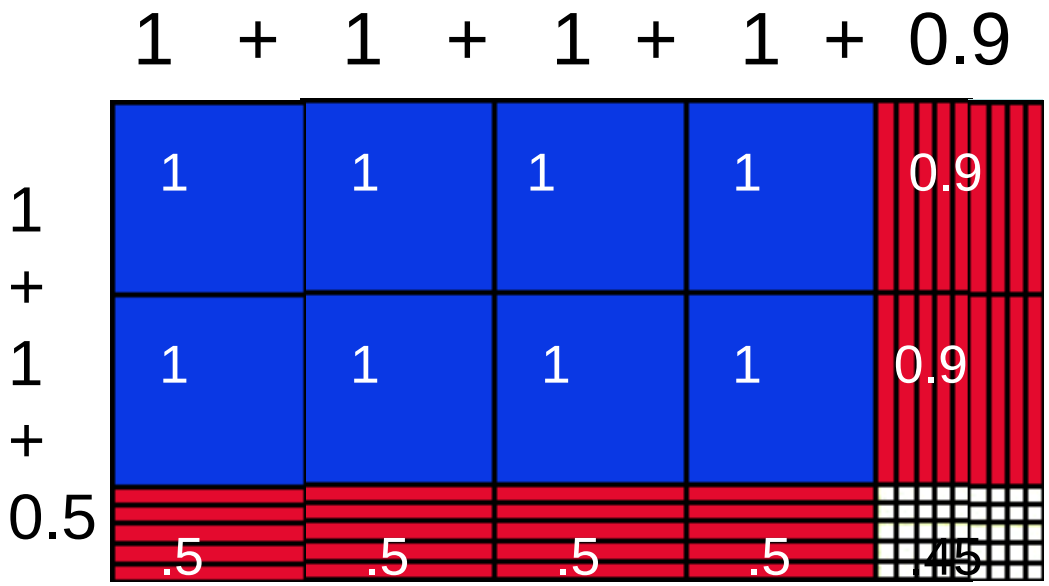
800 + 180 + 200 + 45

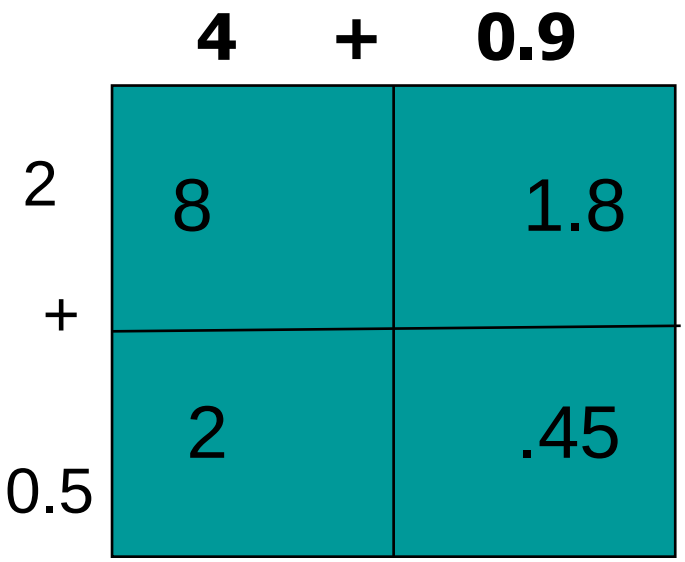
= 1100 + 120 + 5 = 1225

What Does It Look Like?

Decimals … 4.9 x 2.5 = ?





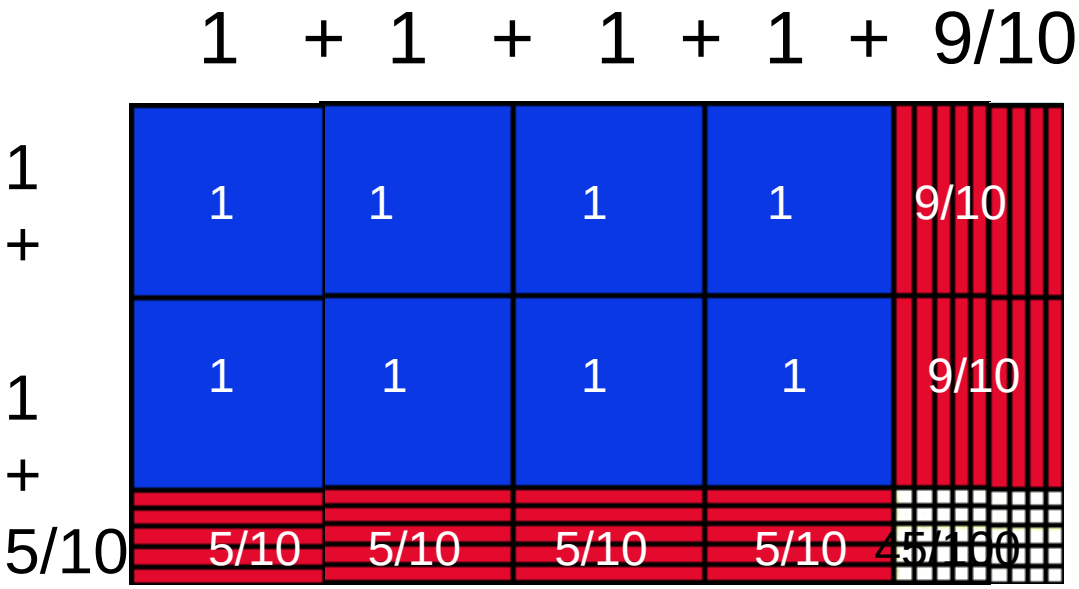


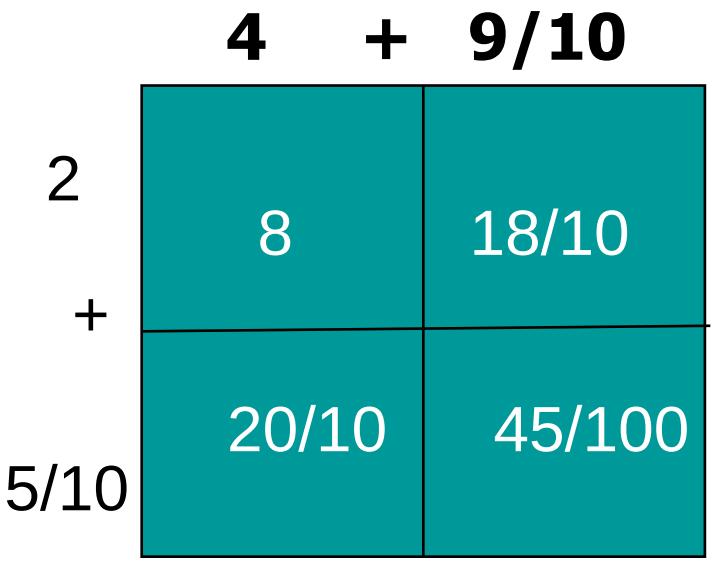
$$8 + 1.8 + 2.0 + .45$$

$$= 11 + 1.2 + .05 = 12.25$$

What Does It Look Like?

Fractions … 4-9/10 x 2-5/10 = ?





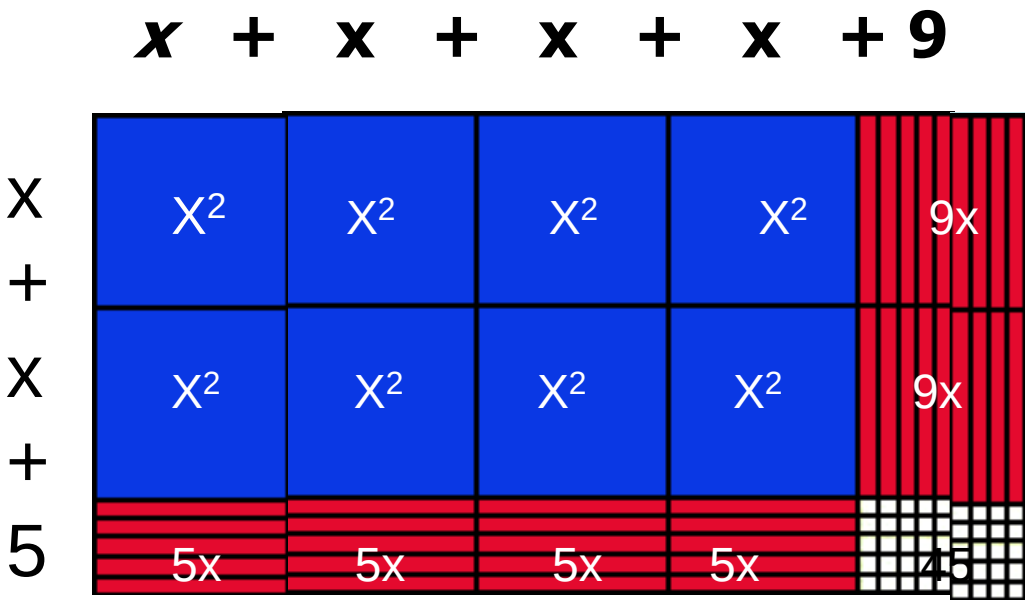
8 + 9/10 + 9/10 + 4(5/10) + 45/100

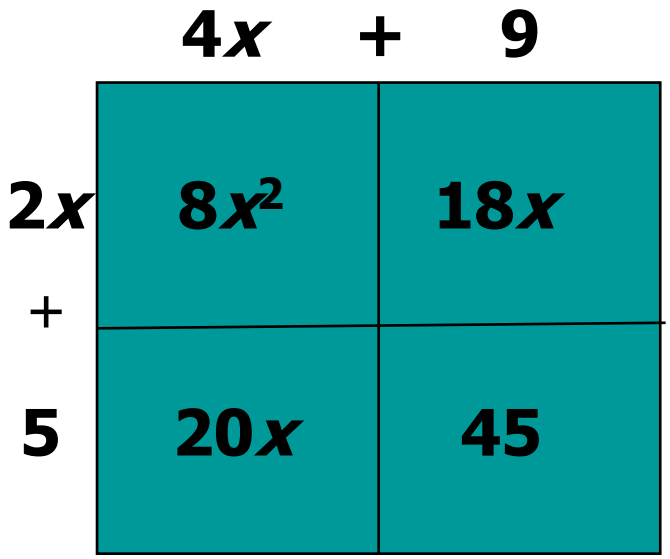
= 8 + 18/10 + 20/10 + 45/100

= 8 + 3 + 8/10 + 45/100 = 12 + 25/100

What Does It Look Like?

Algebraic (4x + 9) (2x + 5) = ?





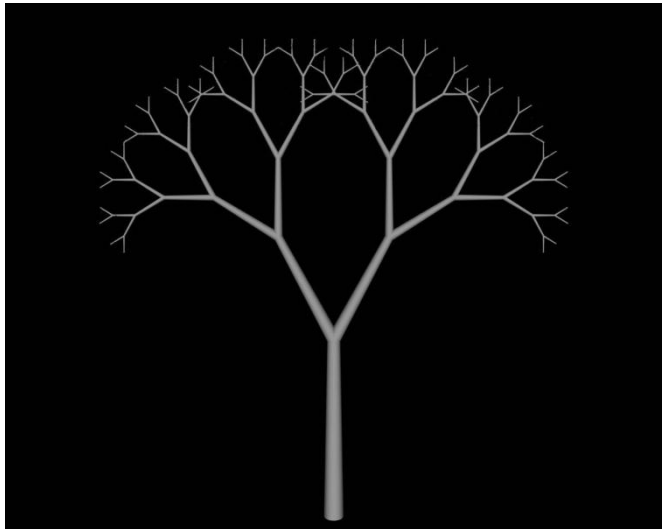
$8x^2 + 38x + 45$

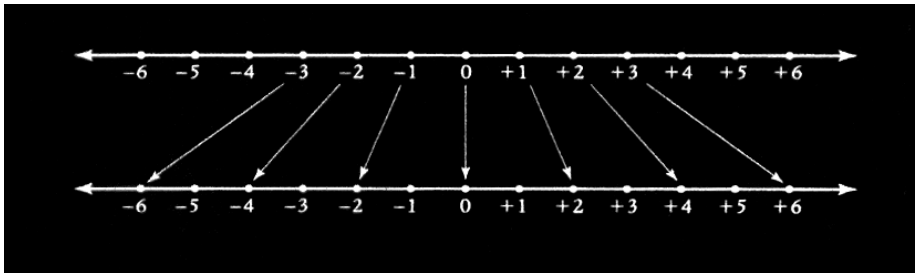
What is Multiplication? (Davis & Simmt, 2005)

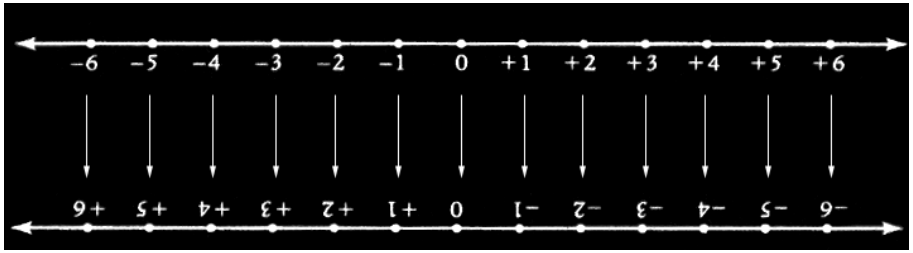
- repeated addition
- grouping process
- sequential folds
- many-layered
- ratios and rates
- grid-generating/array
- dimension-changing
- number-line-stretching or -compressing
- rotating: multiplication by $^{-}1$ rotates the number line by 180˚

$3 \times 2 = 6$

$3 \times t = 3t$

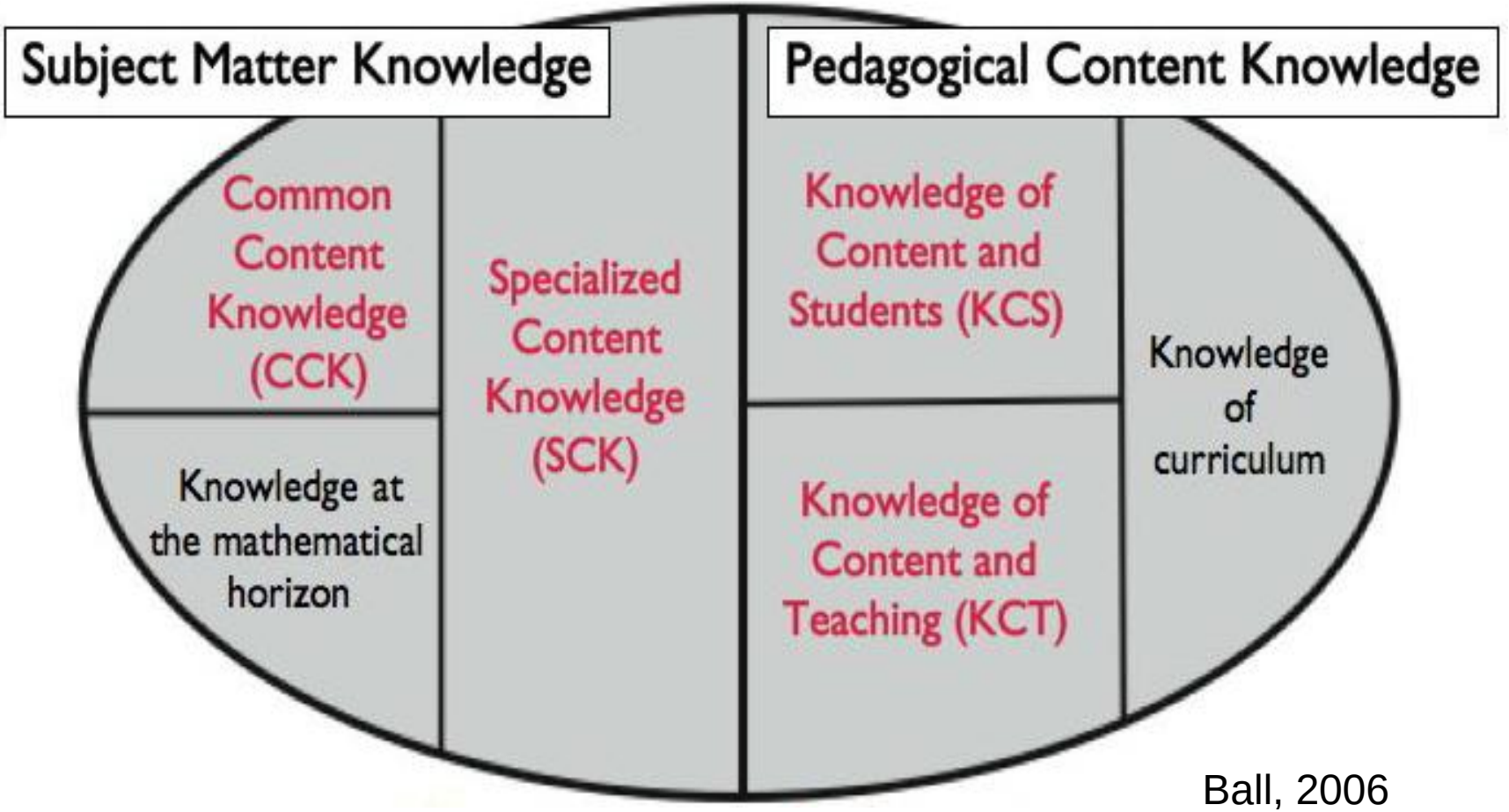






KKZ, 2008

Shulman (1985) and Ball and Bass (2006)



Ball, 2006

Knowledge of Mathematics *for* Teaching

What Does a Teacher Need to Know and Be Able to Do?

BRIDGING PRACTICES

INTERTWINING CONTENT AND PEDAGOGY IN TEACHING AND LEARNING TO TEACH

Deborah Loewenberg Ball
University of Michigan

Subject matter and pedagogy have been peculiarly and persistently divided in the conceptualization and curriculum of teacher education and learning to teach. This fragmentation of practice leaves teachers on their own with the challenge of integrating subject matter knowledge and pedagogy in the contexts of their work. Yet, being able to do this is fundamental to engaging in the core tasks of teaching, and it is critical to being able to teach all students well. This article proposes three problems that would have to be solved to bridge this gap and to prepare teachers who not only know content but can make use of it to help all students learn. The first problem concerns identifying the content knowledge that matters for teaching, the second regards understanding how such knowledge needs to be held, and the third centers on what it takes to learn to use such knowledge in practice.

Work of Mathematics Teaching

- Sequencing of math content and curriculum materials
- Generating and using strategic examples and multiple representations
- Talking mathematics and having students talk
- Understanding and analyzing multiple solutions
- Evaluating the mathematical significance of students' comments and coordinating discussion for learning
- Building correspondence between mathematical ideas, models, and symbols

KKZ, 2008

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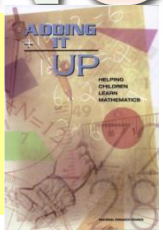
Work of Mathematics Learning

… National Research Council (2001)



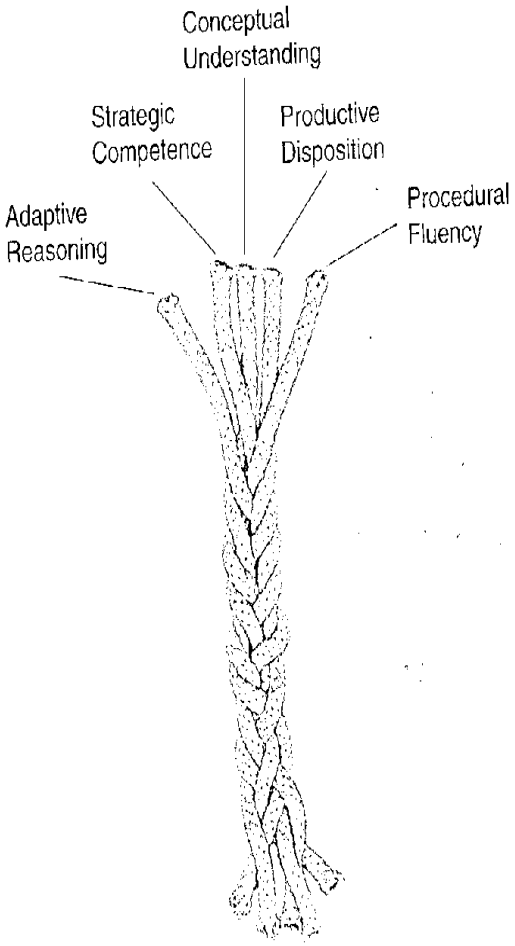
Adaptive Reasoning and Mathematical Communication – thinking deductively and inductively, valuing ambiguity, adapting thought, analyzing, judging, reflecting, explaining, and justifying through multiple representations

Conceptual Understanding - Knowing about number, operations, and relationships among mathematical ideas across the 5 strands

Positive and Productive Disposition - Seeing oneself as being able to do and learn mathematics successfully; viewing mathematics as sensible, useful, and worthwhile

Procedural Fluency - Carrying out procedures flexibly, accurately, efficiently, and appropriately

Strategic Competence - Unpacking the problem, asking questions, making connections, formulating solutions, posing problems, and solving mathematical problems



What are Some Ways to Learn Mathematics *for* Teaching? (Ball, 2005)

- Figure out why procedures work, not just how to do them
- Try to solve problems in more than one way
- Listen to and probe others' thinking, especially when struggling
- Study students' thinking and work
- Talk in class; practise speaking mathematics
- Other ideas?

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What Strategies Did We Use?
- Math-Talk Learning Community

- Give and Get
- Active Listening
- Round Robin
- Turn and Talk